# Lifetime predictions for a ceramic cutting tool<br>material at high temperatures

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Dynamic loading experiments are performed at different loading rates and temperatures in three-point bending with a commercial cutting tool material. Strength data from timedependent failure analysis are combined with Weibull statistics to make lifetime predictions for static loading (STP diagrams). Stress corrosion is concluded to be active from ambient temperatures up to 1100°C. Additionally at high temperatures an energy dissipation process caused by plastic and viscous second phase is activated. Both these mechanisms are interacting. With the application of Weibull statistics, strength data are shown to be strongly influenced by the area of loaded surface.

#### 1. Introduction

The development of ceramic components for any properties and the material requirements and the material application involves the correlation of the material properties and the material requirements, as defined<br>by the in-service conditions (stress, temperature levels). Much of the required information about the tovers), much of the required information about the  $\frac{1}{2}$ .  $\frac{1}{2}$ .  $\frac{1}{2}$ .  $\frac{1}{2}$  $\mu$ aditian capability can be presented on a strengthtime-probability (STP) diagram  $[1, 2]$ . For given conditions of temperature and stress this diagram enables survival probabilities and subset the diagram chapter  $\frac{d}{dt}$  diagrams  $\frac{d}{dt}$  on the existence of  $\frac{d}{dt}$  of  $\frac{d}{dt}$  of  $\frac{d}{dt}$  of  $\frac{d}{dt}$ survival probabilities. The development of the STP diagrams is based on the existence of models to estidiagrams is based on the existence of models to esti-For the include of ceramic material under sitess,  $\Gamma$  and design of a component of  $\Gamma$  component, the dynamic factor of  $\Gamma$ 

have the available for relevant conditions of the conditions  $\alpha$  per and environment. The data on dynamic rangue region to be available for felevalue conditions of temperature and environment. Most of the work with regard to establishing the STP diagrams, however, has been done at room temperature  $[2, 3]$ . Ceramics are increasingly finding use in high-temperature applications like metal cutting and as engine components. There is a need, therefore, to obtain data on the variation of strength with stress rate and establish the STP diagrams, at these high temperatures. If the models for lifetime prediction that are valid at room temperature and not valid at the higher temperatures, the mechanism of crack growth at those temperatures<br>has to be understood to enable the development of models to predict the lifetimes of ceramic material.

### **relationships in ceramics**   $\epsilon$ . Offerflyth probability thire

stress required to propagate small microstructural microstruct flux strength of certainly materials is controlled by the range required to propagate sinan interestructural  $t_{\text{max}}$ , vanation in strength is thus inflerent oue to the range of flaw sizes. In the case of ceramic materials there occurs a degradation in the strength of the material with time. Information is thus required on

- $(1)$  the variation of strength with time, and  $\Gamma$
- (i) the statistical variation in strength,  $\sum_{i=1}^{n}$ (ii) the variation of strength with time, and<br>(iii) the combination of statistics and strength

variation with time under relevant conditions of temperature and environment.

#### 2.1. Statistical variations in strength z. n. oransucur vanations in suchgur<br>Lar

are causarily to propagate the statistic of  $\alpha$  behavior  $\alpha$ material, essentially one or several of the large flaws are caused to propagate. The statistics of fracture strength hence relate to the extreme-value distribution function. One such statistical function due to Weibull [4] has found wide application in describing the strength variation of ceramics. Weibull's function is based on a weakest-link model. He proposed that a body, volume  $V$ , contains a statistical distribution of non-interacting inhomogeneities. Fracture then occurs when any one inhomogeneity reaches critical severity in this volume. The probability of failure then relates to the probability of occurrence of the critical inhomogeneity in the volume  $V$  and is given by

$$
P_{\rm f} = 1 - \exp\left[-\frac{V}{V_0}\left(\frac{\sigma_{\rm f} - \sigma_{\rm u}}{\sigma_0}\right)^m\right] \qquad (1)
$$

where  $\sigma_f$  is the fracture stress and  $\sigma_u$  is the stress below which fracture is assumed to have zero probability (this stress is mostly taken to be zero),  $\sigma_0$  and  $V_0$  are normalizing parameters;  $m$  is a number and is known as the Weibull modulus. It reflects the degree of variability in the strength of the material  $-$  the greater the value of  $m$  is, the less variable is the strength. For ceramics  $m$  varies typically between 5 and 20. Any extrapolation based on the distribution given by Equation 1 is valid only if the material and the mechanism of failure remain the same. The distribution given by Equation 1 can be used to account for effects of changes in geometry and loading.

When  $\sigma_{\rm u}$  can be taken to be zero and no information is available for  $\sigma_0$  and  $V_0$ , the "standardized Weibull distribution" [5] is used:

$$
P_{\rm f} = 1 - \exp\left[-\left(\frac{S}{S_0}\right)^m\right] \tag{2}
$$

S is the fracture stress and  $S_0$  is a normalizing parameter. This function describes the experimental results sufficiently, but lacks any physical basis.

#### 2.2. Time dependence of strength

Time-dependent degradation of strength of the ceramic material occurs due to subcritical crack growth. A stress-dependent chemical interaction between the material and its environment has been identified as the cause of this phenomenon [6]. The time-dependent failure characteristics can be predicted empirically by the stress intensity  $(K_1)$  and crack velocity  $(v = \dot{a})$ data. A typical  $K_1$  against v plot is shown in Fig. 1. There are three regions. In Region I the rate of chemical reaction at the crack front controls the crack growth, described by the experimentally proved empirical equation

$$
v = AK_1^n \tag{3}
$$

In Region II the diffusion and/or flux of the corrosive species to the crack front controls the growth. The Region III shown in Fig. 1 is independent of the corrosive species and describes crack growth by thermal activation, important only at high temperatures. In the presence of a corrosive environment Region II is also negligible for alumina.

If the crack growth rate is described by Equation 3,



*Figure 1* Stress intensity  $K_I$  against crack velocity diagram, schematic.

then from this relation and the relation between the stress intensity  $K_1$ , flaw size a and the applied stress  $\sigma$ given by

$$
K_I = \sigma Y a^{1/2} \tag{4}
$$

where Yis a dimensionless geometric factor, related to specimen dimension, the failure time  $t_{fs}$  for static loading can be obtained as

$$
t_{\rm fs} = \frac{B}{S_{\rm fs}^n} \tag{5}
$$

with

$$
B = \frac{2a_c^{(2-n)/2}}{Y^2A(n-2)}
$$

where  $S<sub>fs</sub>$  is the applied static load and  $a<sub>c</sub>$  is the initial critical defect length.

#### 2.3. SPT and STP diagrams

The stress-probability-time (SPT) diagram can be derived by combining the statistical distribution of Equation 2 and the time-dependent failure analysis as described in the previous section [2, 7, 8]. The basic assumption is that the statistical distribution for static loading is related to that for dynamic loading. It is assumed that the same fracture stress values  $S_{fs}$ ,  $S_{fd}$ respectively, can be obtained in both static and in dynamic loading. The failure time under static load,  $t_{fs}$ , and the failure time under dynamic load,  $t_{fd}$ , are related by [2]

$$
t_{\rm fs} = \frac{1}{n+1} t_{\rm fd} \tag{6}
$$

The distribution for dynamic loading can be related to a normalized distribution for static loading. Hence the static lifetime  $t_{fs}$  as calculated by Equation 6 can be transformed to the corresponding normalized static lifetime  $t_N$ .

$$
\frac{t_{\rm fs}}{t_{\rm N}} = \left(\frac{S_{\rm N}}{S_{\rm fd}}\right)^n \quad \text{for } S_{\rm fd} = S_{\rm fs} \tag{7}
$$

Furthermore, the normalized strength  $S_N$  can be written (from Equations 6 and 7) as

$$
S_{\rm N} = \left(\frac{S_{\rm fd}}{(n+1)\dot{S}t_{\rm N}}\right)^{1/n} S_{\rm fd} \tag{8}
$$

with  $S_{\text{fd}} = \dot{S}t_{\text{fd}}$ . Replacing the value  $S_{\text{fd}}$  from Equation 8 in Equation 2,

$$
P_{\text{fN}} = 1 - \exp\left[\left(\frac{S_{\text{N}}}{S_{\text{ON}}}\right)^{m_{\text{N}}}\right] \tag{9}
$$

where

$$
S_{\text{ON}} = \left(\frac{S_0}{(n+1)St_N}\right)^{1/n} S_0
$$
 and  $m_N = \frac{mn}{n+1}$ 

Equation 9 gives the distribution of the normalized strength values and forms the basis for the SPT and STP diagrams.

This equation can be used to generate SPT and STP plots once the values of  $n$  and the Weibull parameters  $m$  and  $S_0$  for the dynamic measurements are determined. The SPT diagram is a plot of the probability of

TABLE I Specimen composition (material sintered at 1600°C)

Zirconium oxide in non-transform state	$12 \,\mathrm{wt} \,\%$
Tungsten carbide	$2 \times 6$
Cobalt	$< 0.2$ wt $\%$
Aluminium oxide	balance

failure against stress for various lifetimes, and the STP diagram is a plot of lifetime against stress for various probabilities of failure.

#### **3. Experimental methods**

Experiments were conducted to measure the stressrate dependence of the fracture strength (S) at various temperatures. Specimens were fractured in a threepoint bending device at two or three different stress rates. The stress rates differed by two orders of magnitude.

The specimens were made of a commercial cuttingtool material. The composition can be seen in Table I. The experiments were carried out using two different experimental devices. The first set of experiments were done in a resistance furnace, enclosed in a vacuum chamber. The high thermal inertia of this furnace resulted in relatively long holding times of approximately 30 min (heating up excluded). Measurements at 22, 350 and 700 $^{\circ}$ C in air and at 700 and 900 $^{\circ}$ C in a vacuum were carried out with this equipment. The stress rates with this equipment were  $\sim 1.5$  and  $\sim$  150 MPa sec<sup>-1</sup> (Test Series 1). The nominal dimensions of the specimen were  $3 \text{ mm} \times 7.2 \text{ mm} \times 35 \text{ mm}$ with a span of 28 mm. In this series the specimens were loaded in a fiat configuration, i.e. specimen height was 3 mm.

The second device had an induction heating system. With the fast response of the equipment the holding times (including heating up) could be held constant at l0 min for all temperatures and loading rates. Measurements at 700, 900 and 1100°C were carried out with this equipment. The stress rates in this case were  $\sim$  1,  $\sim$  100 and  $\sim$  10 000 MPa sec<sup>-1</sup>. The specimen dimensions in this case were 3.2 mm  $\times$  6 mm  $\times$  35 mm and the span was 24mm. The specimens, in this series, were loaded in an upright position (specimen height  $=$ 6 mm).

#### **4. Results**

The strength values,  $S_{\text{fd}}$ , measured in both the test series, were ranked and the corresponding failure probability  $P_f$  was calculated as

$$
P_{\rm f} = \frac{i}{N+1} \tag{10}
$$

where  $i$  is the rank of the strength value and  $N$  the total number of the strength values. The strength values  $S_{64}$ and the associated failure probability  $P_f$  were fitted to a normalized Weibull distribution by a linear regression of  $\ln \ln [1/(1 - P_f)]$  on  $\ln S_{fd}$ . The Weibull parameters *m* and  $S_0$  obtained from the slope and the intercept of the linear fit, respectively, are given in Table II together with the mean values of strength  $\bar{S}_{\text{fd}}$ for each temperature and stress rate.

The Weibull plots for the strength data at each temperature for the stress rate of  $100 \text{ MPa sec}^{-1}$  are given in Fig. 2 (plots for 900 and  $1100^{\circ}$ C are calculated from Test Series 2). From Fig. 2 it is obvious that the Weibull modulus  $m$  increases with increasing temperature (compare Table II). The variation in the strength is, therefore, lower at higher temperatures.

At each temperature a linear least-squares analysis was done with  $\ln \overline{S}_{6d}$  and  $\ln \overline{S}$  ("mean value plot") to obtain the subcritical crack growth exponent  $n$  [9-11] (see Table III).

Finally, STP design diagrams were calculated from Equation 9 using the appropriate Weibull parameters  $m$ ,  $S_0$  and the subcritical crack growth exponent *n*. Fig. 3 depicts the STP lines that show the lifetime that can be expected at various stress levels for a failure probability of  $0.1\%$  (lines for 900 and  $1100\degree$ C calculated from Test Series 2).

A remarkable difference in the mean values of strength ( $\bar{S}_{\text{fd}}$ , Table II) is observed between the results of test series 1 and 2 at  $700^{\circ}$ C (the temperature at which the measurements in the two sets of experiments overlap). This difference will be discussed in the next section.

#### **5. Discussion**

Table III shows the variation of the subcriticai crack growth exponent n with temperature and environment.

$\tau$ $(^{\circ}C)$	Test Series	$\dot{S}$ (MPa sec <sup>-1</sup> )	$\boldsymbol{N}$	$\bar{S}_{6d}$ (MPa) in air	$\boldsymbol{m}$	$S_0(MPa)$	Correlation coefficient
22		1.7	20	$604 \pm 123$	3.5	680	0.881
22		166.5	20	$646 \pm 134$	4.8	704	0.990
350		1.4	20	$406 +$ 76	5.3	440	0.972
350		136.4	20	$487 +$ 92	5.4	526	0.997
700		0.9	20	344 $\pm$ 65	4.0	383	0.862
700		100.1	20	$397 +$ 70	5.7	428	0.988
700	2	2	13	$462 +$ 71	6.5	493	0.991
700	2	100	13	$550 +$ 59	9.8	576	0.966
700	2	10000	13	572 $\pm$ 97	6.1	613	0.983
900	2		13	$231 \pm$ 20	12.5	240	0.948
900	2	100	13	$238 +$ 25	9.9	250	0.989
900	2	10000	13	$275 +$ 22	13.3	285	0.986
1100	$\overline{2}$		13	166 $\pm$ 28	6.4	178	0.977
1100	2	100	13	197 32 $+$	6.0	211	0.966
1100	2	10000	13	218 $\pm$ 27	8.3	230	0.990

TABLE II Mean fracture strengths and Weibull statistics data at different temperatures



In air, the increase in temperature affects the subcritical crack growth in two ways:

(a) The partial pressure of the water vapour drops, thereby reducing the corrosive effect.

(b) The higher temperature causes the chemical reactions at the crack tip to occur faster, so that stress corrosion is enhanced.

Up to  $700^{\circ}$ C the second effect is dominant, since stress corrosion at the crack tip occurs to a larger extent resulting in lower values of  $n$ . Once the effect of a lower partial pressure becomes dominant, the value of  $n$  increases again with decreasing corrosive influence (Table III).

Negative values of  $n$  are obtained in a vacuum (absence of a corrosive medium) at 700 and  $900^{\circ}$ C, though for a vacuum the  $n$  values are expected to be infinite. The negative values of  $n$  in a vacuum can be attributed to energy dissipation mechanisms at the crack front.

Thus at high temperature two kinds of mechanisms are active in this material:



*Figure 3* STP lines for all measured temperatures, at a probability of failure 0.1%,  $\sigma$  (MPa),  $t_N$  (sec): ( $\Box$ ) 20, (+) 350, ( $\odot$ ) 700, ( $\triangle$ ) 900 and  $(\times)$  1100 $^{\circ}$ C.

*Figure 2* Weibull plots for all measured temperatures, at a loading rate of 100 MPa sec<sup>-1</sup>; ( $\oplus$ ) 20, (\*) 350, (v) 700, (x) 900 and  $(D)$  llo<sup>o</sup> C.

(i) subcritical crack growth by corrosion, and

(ii) an energy dissipation process caused by plastic flow of viscoelastic second phases.

In air the stress corrosion mechanism dominates, with the result that subcritical crack growth occurs, as indicated by positive values of  $n$ . In a vacuum stress corrosion is absent, therefore the energy dissipation processes are dominant. With increasing temperature, the energy dissipation increases resulting in the greater negative value for *n* at 900 $\degree$ C (Table III).

Examination of the fractured surfaces using SEM points to plastic flow of the second phase in the material to be the cause of the energy dissipation. As can be seen from the fractured surfaces at the slowest loading rate of  $1 \text{ MPa sec}^{-1}$  in air at 700, 900 and  $1100^{\circ}$  C (Figs 4a, b and c, respectively), the coating of the grains by the second phase is much greater at  $1100^{\circ}$  C (Fig. 4c) as compared to that at 700 $^{\circ}$  C.

The increase in the value of  $n$  as the temperature increases from 700 to  $900^{\circ}$  C is influenced by both the decreasing corrosive influence of water vapour discussed above and the increasing energy dissipation by the plastic flow of the second phase.

An SEM investigation of the fracture surface of all the specimens broken at 700°C in both the test series (Figs 5 and 6) showed that in most of the specimens, the crack started from the surface (or the edges). This is in violation of the assumption made in arriving at the Weibull function, that the cracks start from the volume of the specimens. Therefore it was

TABLE III Variation of  $n$  with temperature and environment

$\overline{T}$	Test	n	
$(^\circ C)$	<b>Series</b>		
Air			
-22		67	
350		24	
700		32	
700	2	$39 \pm 17$	
900	2	$52 \pm 20$	
1100	$\overline{2}$	$33 \pm 5$	
Vacuum			
700		$-16$	
900		$-128$	







hypothesized that the difference in the strength values between the two test series at 700°C is due to the difference between the loaded surface area in Test Series 1 (specimen held flat) and Test Series 2 (specimen held upright).

The appropriate Weibull function for surface dependence of the crack growth (only the lower surface needs to be considered here) is given by [1, 12]



*Figure 5* Fracture surface appearance, Test Series 1 (flat loading) at 700 $^{\circ}$  C, loading rate 100 MPa sec<sup>-1</sup>: (a) crack starting from volume, (b) crack starting from surface.

*Figure 4* Fracture surface appearance for Test Series 2 at a !oading rate of  $\sim$  1 MPa sec<sup>-1</sup>: (a) 700, (b) 900, (c) 1100°C.

$$
P_{\rm f} = 1 - \exp\left[-\frac{A}{A_0} \left(\frac{\sigma_{\rm f}}{\sigma_0}\right)^m \frac{1}{m+1}\right] \quad (11)
$$

To prove this hypothesis of surface dependence, first the specimens which had cracked from the volume were separated. New standardized distributions were calculated with the remaining specimens and the corresponding mean values of strength and the Weibull parameters  $S_0$  and *m* are given in Table IV. Again, the difference in the mean values of strength between the two test series can be observed. Equating the standardized Weibull function (Equation 2) calculated for the reduced set of specimens and the Weibull function for surface dependency (Equation 11), the following relation is obtained:

$$
\frac{1}{S_0} = \frac{1}{\sigma_0} \left( \frac{A/A_0}{m+1} \right)^{1/m}
$$
 (12)

Writing Equation 12 for both test series, and noting that  $\sigma_0$  and  $A_0$  are constants and  $A_0^{1/m_2}/A_0^{1/m_1} \sim 1$ , the ratio  $S_{0}$ <sub>/</sub> $S_{0}$ <sup>is</sup> given by

$$
S_{0_2}/S_{0_1} = [(m_2 + 1)/A_2]^{1/m_2}/[(m_1 + 1)/A_1]^{1/m_1}
$$
 (13)

Equation 13 thus provides a way to account for the difference in the area of the loaded surface and as can



*Figure 6* Fracture surface appearance, Test Series 2 (upright loading) at 700 $^{\circ}$ C, loading rate 100 MPa sec<sup>-1</sup>: (a) crack starting from volume, (b) crack starting from surface.

Weibull functions at 700°C Test  $\dot{S} (\text{MPa sec}^{-1})$   $N$   $\bar{S}_{\text{fd}} (\text{MPa})$   $m$   $S_0 (\text{MPa})$  Correlation Series coefficient  $1/m$  $A$  (mm<sup>2</sup>)  $m +$ Series coefficient  $\left(\frac{-\overline{A}}{A}\right)$  $1 \hspace{1.5cm} 0.9 \hspace{1.5cm} 12 \hspace{1.5cm} 340 \pm 68 \hspace{1.5cm} 4.67 \hspace{1.5cm} 368 \hspace{1.5cm} 0.830 \hspace{1.5cm} 201.6 \hspace{1.5cm} 0.466$ 2 2 8 452  $\pm$  78 5.56 486 0.973 76.8 0.642  $\frac{100.1}{0.568}$  13  $\frac{411 \pm 71}{1.5}$  5.95  $\frac{441}{1.5}$  0.973 201.6 0.568 2 100 6  $558 \pm 43$  12.3 578 0.918 76.8 0.867

TABLE IV Mean fracture strengths and Weibull statistics data, loaded areas and coefficients in the Weibull exponents for reduced

be seen from Table V, though the distributions are obtained from a small number of specimens, the agreement of the measured parameters with the above relation is obvious.

Therefore, the difference in strength values between the two test series at  $700^{\circ}$  C is caused by the difference in the loaded areas, which was a consequence of the difference in the loading configuration in the two test series.

#### **6. Concluding remarks**

It is clear that at some temperature the stress corrosion mechanism will cease to be active, as the environment responsible for corrosion would have dispersed. Various mechanisms have been proposed to describe the subcritical crack growth mechanism, based on diffusional theories [13, 14] and plastic flow theories [15-17]. While quantitative models exist for these mechanisms, they are not of much use in establishing the design diagrams. This is because they involve too many material parameters. As the failure time is very sensitive to stress, all theories seem to fit the data and it is not possible to distinguish between the various models based on their agreement with the data in the literature [18].

While a comprehensive model to describe the subcritical crack growth mechanism at high temperatures seems far away, empirical models can be developed to make lifetime predictions at these temperatures. The problems due to non-linearity of the loaddisplacement relation can be eliminated by using a different parameter to describe the relationship between the load and the crack velocity. Two such parameters have been proposed in the literatme. The J-integral based on the energy concept [19] is applicable to subcritical crack growth due to elastoplastic reactions. At very low loading rates, however, in addition to the crack growth due to viscous (elastoplastic) reactions, crack growth due to creep also occurs. In such cases the power integral  $C^*$  [20] may be a suitable parameter. For the material under investigation the J-integral would be best suited to describe the relationship between load and crack velocity and hence enable more accurate lifetime predictions.

TABLE V Comparison of coefficients in the Weibull exponents for reduced Weibull functions at 700°C

S(MPa)	$S_{0}^{2}/S_{0}^{2}$	$1/m_1$ 1/m <sub>2</sub> $\frac{1}{2}$ $\frac{m_1 + 1}{m_2 + 1}$ $\left(\frac{m_2+1}{A_2}\right)^{n}$
$\sim$ 1	1.32	1.38
$\sim$ 100	1.31	1.52

#### **7. Summary**

STP diagrams have been established for a commercial ceramic cutting-tool material, at temperatures ranging from ambient to  $1100^{\circ}$  C. Two mechanisms have been found to be active. In air the stress corrosion mechanism, on which the STP concept is based, is dominant even at  $1100^{\circ}$  C. In a vacuum at high temperatures an energy-dissipating mechanism has been found to be active. The interaction of the two mechanisms is responsible for the subcritical crack growth at the higher temperatures. These STP design diagrams enable lifetime predictions to be made at operating temperatures for the actual component, based on tests conducted on simple geometries and loading configurations.

Another problem to be considered is the systematic influence of the experimental procedure on the statistics of the data. Such influences have to be accounted for, as has been done for the influence of loaded surface area on the strength data.

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